A Note on *N*-Way Hybrid Power Dividers

An *N*-way hybrid power divider having excellent isolation between ports was described by Wilkenson[1] and subsequently by Peterson.[2] The equivalent network is shown in Fig. 1. Wilkenson and Peterson determined the values of transmission-line impedances and resistances at midband for conditions of input match and perfect isolation. The theoretical performance of this device at other frequencies was not given. This communication analyzes the isolation and the input SWR as a function of frequency. An interesting extension of odd and even mode methods, normally used to analyze

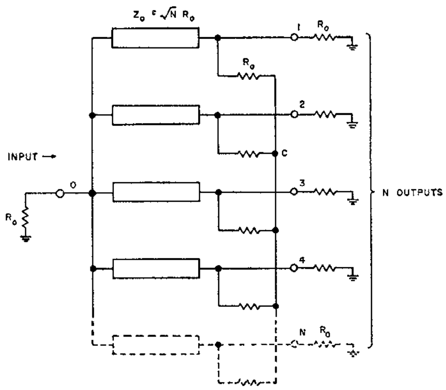

Fig. 1—*N*-way hybrid power divider.

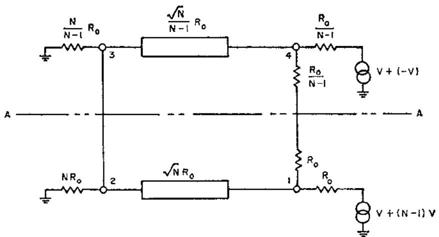

Fig. 2—Equivalent four-port network.

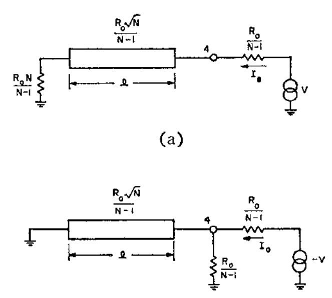

Fig. 3—Odd and even mode networks.

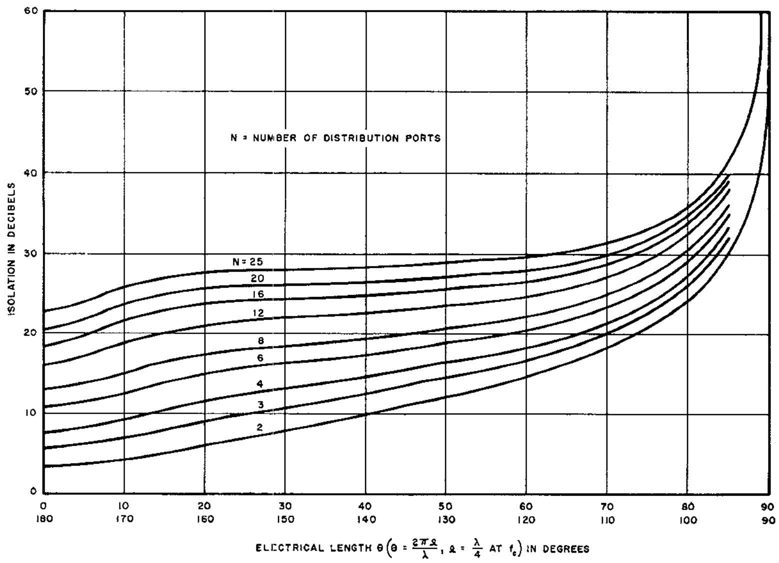

Fig. 4—Output isolation of *N*-way hybrid power divider.

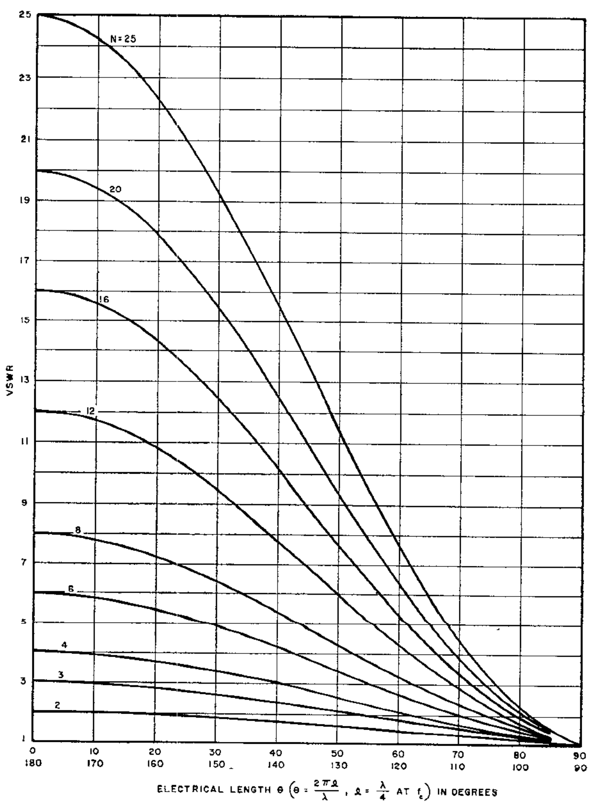

Fig. 5—VSWR of *N*-way hybrid power divider.

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[1] E. Wilkenson, "An *N*-way hybrid power divider," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-8, pp. 116–118; January, 1960.

[2] R. W. Peterson, "*N*-terminal power divider," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-9, p. 571; November, 1961.

symmetrical four-port networks,[3,4] is applied to the N-way symmetrical power dividers.

The N-way power divider consists of N-transmission lines of characteristic impedance $\sqrt{N}R_0$ that are chosen to be a quarter-wavelength long at the desired mid-band frequency. The generator and distribution ports are each terminated by resistors R_0. A lumped internal resistor of value R_0 is connected from each of the distribution ports to a common point C.

To determine the isolation of one distribution port with respect to any other port, an equivalent four-port version of the device is analyzed (See Fig. 2). If a voltage generator is placed at one of the distribution ports, the total power (P_4) delivered to the other $N-1$ ports is dissipated in an equivalent resistor, $R_0/(N-1)$. The power delivered to any one port is $P_t = P_4/(N-1)$. The generator resistance R_0 is split into two parallel resistors of value NR_0 and $N/(N-1)R_0$. All $N-1$ internal resistances and transmission-line impedances are effectively in parallel and are thereby reduced by a $1/(N-1)$ factor. By choosing an open-circuit generator voltage of $NV = V + (N-1)V$ and placing generator voltages of $+V$ and $-V$ in series with the load at port 4, it is possible to analyze this network by the superposition of even and odd modes. The even mode corresponds to an open circuit at plane $A-A$ with excitations of $+V$ in series with the port 4 load. The odd mode corresponds to a short circuit at plane $A-A$ with excitation of $-V$ in series with the port 4 load. This differs from the methods given in Jones and Bolljahn[3] and in Reed and Wheeler[4] in that the even mode is achieved with a voltage at port 1 that is unequal in magnitude to that at port 4, that is, $V_1 = (N-1)V$ and $V_4 = -V$.

The even mode case is shown in Fig. 3(a). The current flowing through the resistor $R_0/(N-1)$ is

$$I_e = \frac{V/R_0}{\dfrac{1}{N-1} + \dfrac{\sqrt{N}}{N-1}\,\dfrac{\sqrt{N}\cot\theta + j}{\cot\theta + j\sqrt{N}}} \quad (1)$$

where $\theta = (2\pi/\lambda)l$; l is chosen to make $\theta = \pi/2$ at the design-center frequency f_c.

The odd mode case is shown in Fig. 3(b). The corresponding odd mode current is

$$I_0 = -\frac{V(N-1)}{R_0}\,\frac{(\cot\theta + j\sqrt{N})}{\cot\theta + j2\sqrt{N}}. \quad (2)$$

The total current is the sum of I_e and I_0. The power delivered to this load is

$$P_t = |I_e + I_0|^2 \frac{R_0}{N-1}. \quad (3)$$

The power delivered to any one port P_t is

$$P_t = \frac{|I_e + I_0|^2 R_0}{(N-1)^2}. \quad (4)$$

The isolation is defined as

$$I = 10 \log \frac{P_a}{P_t} \text{ db} \quad (5)$$

where P_a is the available power at port 1;

$$P_a = \frac{N^2V^2}{4R_0}. \quad (6)$$

Substituting (1) and (2) in (4) gives the desired value of P_t. Substituting this expression together with (6) in (5) gives the following desired result:

$$I = 10 \log \frac{N^2}{4\left|\dfrac{\cot\theta + j\sqrt{N}}{(N+1)\cot\theta + j2\sqrt{N}} - \dfrac{\cot\theta + j\sqrt{N}}{\cot\theta + j2\sqrt{N}}\right|^2} \text{ db}. \quad (7)$$

This result has been plotted in Fig. 4 as a function of θ with N as a parameter. These curves are useful in predicting the bandwidth of an N-way divider for a specified isolation.

The VSWR, at the input to the power divider (port O in Fig. 1), is obtained from

$$r = \frac{1 + \dfrac{|Z_{in} - R_0|}{|Z_{in} + R_0|}}{1 - \dfrac{|Z_{in} - R_0|}{|Z_{in} + R_0|}} \quad (8)$$

where

$$Z_{in} = \frac{R_0}{\sqrt{N}}\,\frac{1 + j\sqrt{N}\tan\theta}{\sqrt{N} + j\tan\theta}. \quad (9)$$

In obtaining this expression for Z_{in}, the internal resistors are effectively removed from the circuit because no current flows in these resistors when the device is excited at port O. The input VSWR is plotted in Fig. 5 as a function of θ for various values of N. These curves are useful in determining the power distributed to each of the N ports. This power is reduced from the midband loss ratio N by the input reflection loss. The distribution loss at any frequency is therefore determined by

$$L_D = 10 \log \frac{N}{1 - \left(\dfrac{r-1}{r+1}\right)^2} \text{ db}. \quad (10)$$

The isolation of an N-way hybrid power divider has been analyzed as a function of frequency using an extension of odd and even mode techniques previously used for four-port networks. The input VSWR characteristics have been determined using a more conventional method. It is believed that the analysis described herein will be applicable to other N-port symmetrical networks.

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[3] J. T. Jones and E. M. T. Bolljahn, "Coupled-strip-transmission-line filters and directional couplers," IRE Trans. on Microwave Theory and Techniques, vol. MTT-4, pp. 75–81; April, 1956.

[4] J. Rede and G. J. Wheeler, "A method of analysis of symmetrical four-port networks," IRE Trans. on Microwave Theory and Techniques, vol. MTT-4, pp. 246–253; October, 1956.